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BUBBLE CONDENSATION WITH NON-HOMOGENEOUS DISTRIBUTION OF NON-CONDENSABLES

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NOMENCLATURE

C_p, specific heat;
F_1, $F_1(\beta_f^H)$, function of β_f^H, equation (20);
F_2, $F_2(\beta_f^P)$, function of β_f^P, equation (24);
G^*, ratio, liquid to vapor density, $\rho_L/\rho_v{}^*$;
Ja, Jacob number $(\rho C_p \Delta T/\lambda\rho_v)$;
k, thermal conductivity;
K_v, velocity factor, modified potential flow;
P^*, total system pressure;
P_g, partial pressure, non condensible;
P_v, partial pressure vapor;
Pe, Péclét number $(U_\infty 2R/\alpha)$;
Pr, Prandtl number $(\mu C_p/k)$;
Nu, Nusselt number $(h2R/k)$;
q, heat flux [Btu/hft^2];
q_0, heat flux, potential flow [Btu/hft^2];
R, radius of bubble;
R_0, initial radius of bubble;
R_f, final radius of bubble;
$\hat{R}$, specific gas constant;
r, radial coordinate;
T^*, saturation temperature corresponding to P^*;
T_w, bubble wall temperature;
T_∞, approach temperature, surrounding liquid;
ΔT, temperature difference, $T^* - T_\infty$;
t, time;
U, velocity of rise;
α, thermal diffusivity;
β, dimensionless radius, R/R_0;
β_f, final dimensionless radius, R_f/R_0;
λ, latent heat;
ρ, density, continuous phase;
ρ_L, density, condensate;
ρ_v, density, vapor;
τ, dimensionless time, Fourier number $(=\alpha t/R_0^2)$;
$\hat{\tau}$, dimensionless time for collapsing bubble $(JaPe^{\frac{1}{2}}\tau)$.

Subscripts
f, final;
0, initial;
w, at the wall.

Superscripts
H, homogeneous distribution;
P, parabolic distribution.

INTRODUCTION

WITTKE and Chao [1] and Isenberg and Sideman [2] presented numerical solutions for unsteady state bubble collapse; the former for a single component (steam–water) system and the latter for a two component (pentane–water) system. These systems differ since the condensate in a single component bubble merges with the surrounding liquid, while the condensate in the two component system remains within the confines of the bubble walls. More recently, Sideman *et al.* [3] presented an approximate, quasi-steady state, analytical solution for bubble collapse in two-component, 3-phase systems. The solution is general, conveniently reducing to a solution for a single component

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system, and is in agreement with the experimental data for pentane–water, pentane–pentane and steam–water systems.

All past solutions are based on the assumption that the non-condensables are homogeneously distributed in the confined space of the condensing bubble. This assumption may be reasonable for a single phase bubble where a potential flow field is assumed and the internal circulation associated with the moving boundary may approach a completely mixed vapor–gas condition. However, this is definitely not the case for the two component system where the tangential motion of the wall may be drastically hindered by the condensing film on the bubble's wall.

It is the object of this communication to present a complete and general solution for bubble condensation, taking into account the concentration gradients inside the condensing bubble.

For expediency as well as for clarity, and with little loss of accuracy, the approximate quasi-steady state analytical solution will be presented here. To avoid confusion, and to account for the presence of inerts, the bubbles in the single and two-components systems are hereby denoted "single-phase" and "two-phase" bubbles, respectively.

THEORETICAL

For heat transfer controlled bubble collapse at relatively high (> 1000) Péclét numbers the quasi-steady state assumption implies that the radial wall velocity is negligible as compared with the translatory motion of the bubble. In other words, the temperature field around the bubble attains a steady state instantaneously, for every radius of the bubble. The heat flux is then given by

$$q = q_0 \cdot (K_v)^{\frac{1}{2}} \tag{1}$$

where q_0, the average heat flux in a potential flow field, here related to the instantaneous radius R, is given by

$$q_0 = \frac{k(T_w - T_\infty)}{R\pi^{\frac{1}{2}}}\left(\frac{2RU_\infty}{\alpha}\right)^{\frac{1}{2}} \tag{2}$$

and K_v, the velocity factor by which the potential flow solution for flow around a sphere is "transformed" to yield the average heat flux, equivalent to that which a laminar flow field solution would have given, is given by [2, 3],

$$K_v = 0{\cdot}25\, Pr^{-\frac{1}{3}}. \tag{3}$$

Equating equation (1) with the average heat flux being removed from the collapsing bubble, i.e. $-\lambda\rho_v\dot{R}$, and defining

$$\beta \equiv \frac{R}{R_0} \qquad \hat{t} = JaPe^{\frac{1}{2}}\tau \qquad \theta_w = \frac{T_w - T_\infty}{T^* - T_\infty}$$

yields:

$$\frac{d\beta}{d\hat{t}} = -\left(\frac{K_v}{\pi\beta}\right)^{\frac{1}{2}}\theta_w; \qquad \beta = 1 \text{ at } \hat{t} = 0. \tag{4}$$

Note that the Jacob number is defined here in terms of the properties of the cooling fluid and the saturation temperature T^* corresponding to the pressure of the system p^*.

In the absence of non-condensables $T_w = T^*$, the wall temperature is identical with the saturation temperature, and $\theta_w = 1$. Integration of equation (4) then yields

$$\beta = \left[1 - \frac{3}{2}\left(\frac{K_v}{\pi}\right)^{\frac{1}{2}}\hat{t}\right]^{\frac{2}{3}} \tag{5}$$

or

$$\hat{t}_0 = \tfrac{2}{3}(\pi/K_v)^{\frac{1}{2}}(1 - \beta^{\frac{3}{2}}). \tag{5a}$$

In the presence of non-condensables, $T_w \neq T^*$. The partial pressure of the inert gas increases as the bubble contracts, simultaneously reducing the partial pressure of the vapors, until, as $T_w \to T_\infty$ condensation stops. At this point $\beta = \beta_f$.

Integration of equation (4), accounting for the presence of non-condensables, requires explicit expressions relating θ_w to the inerts concentration and the instantaneous radius of the bubble. However, the inerts concentration is clearly related to the final bubble diameter, β_f. Since the latter may be readily available experimentally, it is deemed advantageous to express θ_w in terms of the instantaneous and final dimensionless radii. Obviously, the value of θ_w as well as β_f must depend on the distribution of the inerts within the bubble.

(a) *Homogeneous distribution of non-condensables*

For this case the final radius is related to y_0, the initial mole fraction of the non-condensables in the bubble by [2]:

$$\beta_f^H = \left[\frac{\hat{R}T^{*2}y_0}{\lambda(T^* - T_\infty)} + \frac{1}{G^*}\right]^{\frac{1}{3}}: \qquad G^* \equiv \rho_L/\rho_v^* \tag{6}$$

where $\hat{R}$ is the gas constant. The superscript H denotes the homogeneous case. The term $1/G^*$ is due to the accumulation of the condensate within the confines of the two-phase bubble. For a single-phase bubble $1/G^*$ vanishes and in the absence of non-condensables $y_0 = 0$ and $\beta_f = 0$.

In terms of β_f^H and β, the dimensionless wall temperature is given by (2):

$$\theta_w^H \simeq \frac{P_{v,w} - P_{v,\infty}}{P^* - P_{v,\infty}} = \frac{\beta^3 - \beta_f^{H^3}}{\beta^3 - 1/G^*} \tag{7}$$

where P_v is the partial pressure of the vapor and P^* is the total system pressure. Second indices w and ∞ denote the partial pressures corresponding to T_w and T_∞, respectively. Note that $1/G^*$ vanishes for a single phase bubble.

Introducing equation (7) into (4) and integrating yields

$$\hat{t}^H = \hat{t}_0 + \hat{t}_1 \tag{8}$$

where $\hat{t}_1$ is given by

$$\hat{t}_1 = \left(\frac{\pi}{K_v}\right)^{\frac{1}{2}}\frac{\beta_f^{H^3} - (1/G^*)}{3\beta_f^{H\frac{3}{2}}}\ln\frac{(1 - \beta_f^{H\frac{3}{2}})(\beta^{\frac{3}{2}} + \beta_f^{H\frac{3}{2}})}{(1 + \beta_f^{H\frac{3}{2}})(\beta^{\frac{3}{2}} - \beta_f^{H\frac{3}{2}})}. \tag{9}$$

For a single phase bubble $K_v = 1$ and $1/G^* = 0$.

(b) *Non-homogeneous concentration distribution*

A parabolic concentration profile of the inerts inside the bubble is assumed:

$$y = a_0 + A_1 r + A_2 r^2 \tag{10}$$

where y is the instantaneous spacial concentration and r is the radial coordinate.

The boundary conditions are

$$y = y_0 \quad \text{at} \quad r = 0 \tag{11}$$

$$D\frac{\mathrm{d}y}{\mathrm{d}r} = 0 \quad \text{at} \quad r = 0 \tag{12}$$

$$D\frac{\mathrm{d}y}{\mathrm{d}r} = 0 \quad \text{at} \quad r = R \tag{13}$$

Equations (11) and (12) imply that, within the relatively short condensation period, the concentration in the center of the bubble remains constant, at the initial concentration of the emerging, still uncondensed, bubble. Equation (12) is self evident due to symmetry. Equation (13) states that the inert gas remains within the confines of the bubble, with no loss due to gas diffusion into the surrounding liquid.

Neglecting the volume occupied by the liquid in the two phase bubble ($1/G^* \simeq 0{\cdot}005$) the conservation of the inerts is written as

$$\tfrac{4}{3}\pi R_0^3 y_0 = \int_0^R y(r)\, 4\pi r^2 \, \mathrm{d}r. \tag{14}$$

Note that since $R = f(t)$, y is also a time dependent variable. Equation (10) becomes

$$y = y_0 \left[1 + \frac{5}{3}\left(\frac{1-\beta^3}{\beta^3}\right)\frac{r^2}{R^2}\right] \tag{15}$$

which reduces to $y = y_0$ at $r = 0$. For $r = R$ equation (15) reduces to

$$Y_{w,0} = Y_0: \qquad \beta = 1 \tag{16a}$$

$$y_w = y_0\left[1 + \frac{5}{3}\left(\frac{1-\beta^3}{\beta^3}\right)\right]: \qquad 1 > \beta > \beta_f \tag{16b}$$

and

$$y_{w,f} = y_0\left[1 + \frac{5}{3}\left(\frac{1-\beta_f^{p^3}}{\beta_f^{p^3}}\right)\right]: \qquad \beta = \beta_f \tag{16c}$$

$y_{w,f} > y_w > y_0$, consistent with physical reality. Note that β_f^p denotes the final (dimensionless) radius in the non-homogeneous case.

The relationship between the initial concentration and final (dimensionless) radius is derived in a manner similar to that leading to equation (6) utilizing Dalton's law and Clausius–Clapeyron's equation. Here, however, the subscript w denotes a point value, namely 'at the wall' (and only at the wall). Thus, the ratio of the initial and final partial pressures of the inerts P_g, near the wall is given by

$$\frac{P_{g,w,0}}{P_{g,w,f}} \simeq \frac{\hat{R}T^{*2}\, y}{\lambda(T^* - T_\infty)}. \tag{17}$$

Also, since the initial concentration is uniform throughout the bubble,

$$\frac{P_{g,w,0}}{P_{g,w,f}} = \frac{y|_{r=R,\beta=1}}{y|_{r=R,\beta=\beta_f}} = \frac{y_0}{y_{w,f}}. \tag{18}$$

Introducing equations (16) and (17) into (18) and utilizing β_f^H as given by equation (6) yields,

$$\beta_f^P = \beta_f^H\, F_1(\beta_f^H) \tag{19}$$

where

$$F_1(\beta_f^H) = \left[\frac{5}{3 + 2(\beta_f^H)^3}\right]^{\frac{1}{3}}. \tag{20}$$

Equation (19) is exact for a single-phase bubble. For a two-phase bubble, equation (19) will yield good results at the initial condensation stages where $1/G^*$ in equation (6) is negligible. This is particularly correct at large values of $Y_0/(T^* - T_\infty)$.

The relationship presented in equation (7) between θ_w and the partial pressures of the vapor (and gas) in the bubble holds for this case too. However, $P_{v,w}$ in equation (7) represents the homogeneous partial pressure in the drop corresponding to the homogeneous temperature T_w. In the non-homogeneous case $P_{v,w}$ represents the partial pressure of the vapor actually at the wall temperature, T_w (point value). Similarily, $P_{v,\infty}$ which in equation (7) represents the homogeneous partial pressure in the bubble at its final size β_f^H and temperature T_∞, must be replaced by $P_{v,w,f}$ which denotes the final partial pressure of the vapor at the wall, where and when $T_w = T_\infty$. Thus,

$$\theta_w^P = \frac{T_w - T_\infty}{T^* - T_\infty} \simeq \frac{P_{v,w} - P_{v,w,f}}{P^* - P_{v,w,f}}. \tag{21}$$

Introducing $P_{v,w} = P^* - P_{g,w}$; $P_{v,w,f} = P^* - P_{g,w,f}$, and $P_g/P^* = y$ yields

$$\theta_w^P = 1 - \frac{P_{g,w}}{P_{g,w,f}} = 1 - \frac{Y_w}{Y_{w,f}}. \tag{22}$$

Combining equations (16) and (22) yields

$$\theta_w^P = \frac{1}{F_2}\,\frac{\beta^3 - \beta_f^{p^3}}{\beta^3 - 1/G^*} \tag{23}$$

where

$$F_2 \equiv F_2(\beta_f^p) = 1 - \tfrac{2}{5}\beta_f^{p^3} \equiv 1 - \tfrac{2}{5}\beta_f^H\; F_1^3. \tag{24}$$

Note that equation (23) represents an exact solution of equation (16) and (22) only for a single phase bubble, where $1/G^* = 0$. However, equation (21) applies to the two-phase bubble as well and, by analogy to equation (7), the term $1/G^*$ is introduced as the best approximation for the two-phase bubble.

Introducing equation (23) into (4) and integrating yields

$$\hat{t}^P = F_2\,[\tau_0 + \hat{t}_1^P] \tag{25}$$

or, in terms of β and β_f

$$\hat{t}^P = F_2\,(\beta_f^P)\,[\hat{t}_0(\beta) + \hat{t}_1\,(\beta, \beta_f^P)] \tag{26}$$

where $\hat{t}_1$ is calculated by equation (9), with $\beta_f^P = F_1\beta_f^H$ now replacing β_f^H.

RESULTS AND CONCLUSIONS

A plot of $F_1\,(\beta_f^H)$ and β_f^P vs. β_f^H is presented in Fig. 1. As can be seen from Fig. 1(a) and equation (20), $F_1\,(\beta_f^H)$ is, with an error of less than 4 per cent, practically constant ($\simeq 1{\cdot}18$) for $\beta_f^H < 0{\cdot}4$. Note that for the same y_0 $\beta_f^P > \beta_f^H$, since $\beta_f^H \leq 1$ and $F_1\,(\beta_f^H) \geq 1$.

It is interesting to elucidate the relationship between θ_w^P, equation (23), and θ_w^H, equation (7). As seen in Fig. 1(c) $F_2\,(\beta_f^P)$ decreases with β_f, going from 1 for $\beta_f = 0$ to 0·6 for $\beta_f = 1$. However, in practice $\theta_w^P < \theta_w^H$ and condensation stops faster for the non-homogeneous case. Clearly, the effect of β_f^P is overriding that of $1/F_2$.

For the same initial inerts content in the bubble, a concentration gradient inside the bubble retards condensation more than when the inerts are uniformly distributed in the bubble. This is demonstrated in Figs. 2 and 3 where β is plotted against the (dimensionless) time. For identical initial operating conditions, the instantaneous radius β after any time interval will be lower for the homogeneous bubble than for the comparable bubble with a concentration build-up at the wall. The effect of non-homogeneity is small in the initial condensation stages but increases as β decreases. The larger the initial inert concentration (hence larger β_f) the larger will be the effect of non-homogeneity. Conversely, for low initial y_0, the correction for the concentration gradient is small and is significant only at the last stages of the condensation process.

The derivation presented are exact for single phase bubbles where $1/G^*$ is identically zero. However, since $1/G^*$ which represents the volume occupied by the immiscible condensate within the confines of the two-phase bubble is quite small ($\approx 1/200$), the solution for two-phase bubbles is quite accurate for large values of β. The solution is therefore accurate at the initial condensation stages, where the volume occupied by the condensate is negligible. Obviously, the

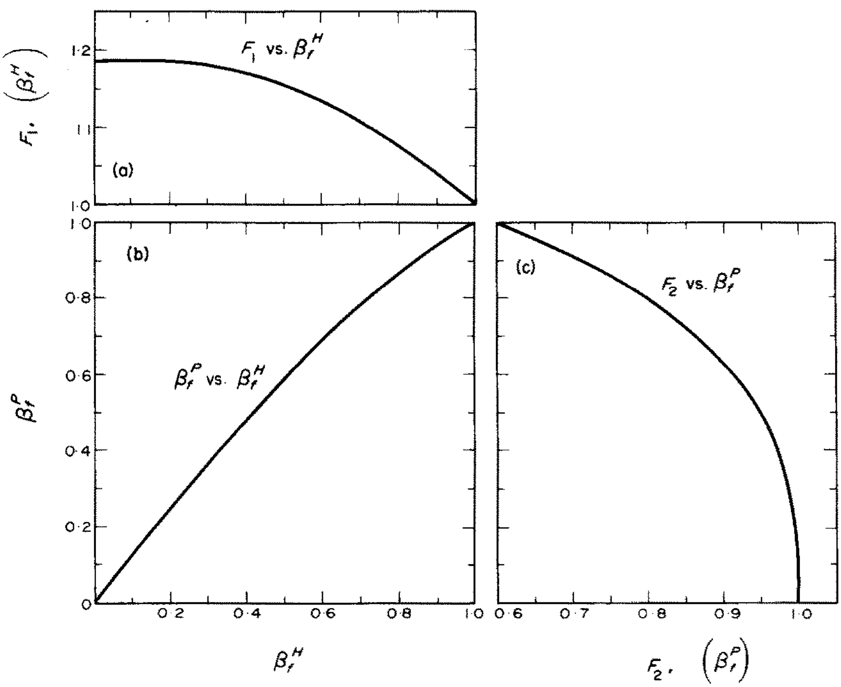

FIG. 1. (a) $F_1(\beta_f^H)$ vs. β_f^H; (b) β_f^P vs. β_f^H; (c) $F_2(\beta_f^P)$ vs. β_f^P.

accuracy improves over the whole condensation process as β_f, or rather $(y_0/\Delta T)$, increases.

Finally, compare the condensation rates of pentane in the single and two component systems. At identical $\hat{\tau}$ the single-phase bubble in the potential flow field will collapse faster than the two-phase bubble in the modified, restricted, potential field. However, a comparison based on identical bubble radii and temperature driving force, shows that the condensation of a pentane bubble in water is about 50 per cent faster than in pentane. This conclusion, experimentally verified [3], indicates one of the practical advantages inherent in utilizing 3-phase heat exchanges.

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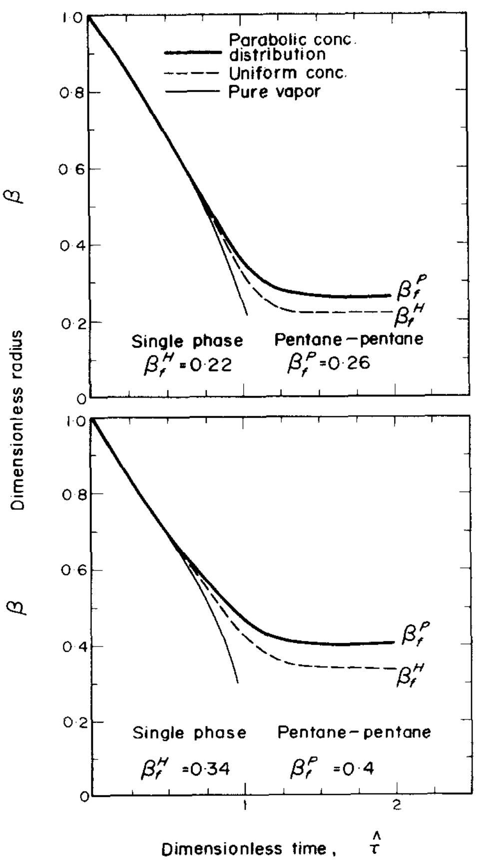

Fig. 2. Dimensionless time vs. dimensionless radius for a pentane bubble condensing in subcooled pentane.

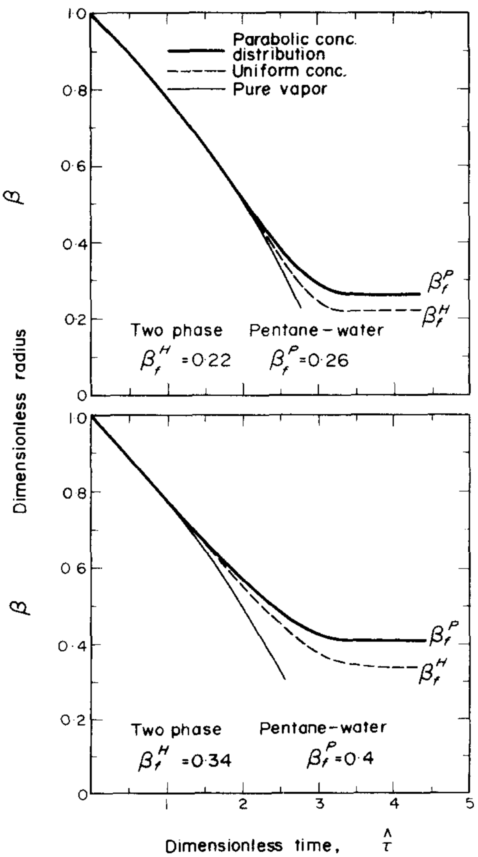

Fig. 3. Dimensionless time vs. dimensionless radius for a pentane bubble condensing in water.